Solving multi-objective Intuitionistic linear programming using Triangular Intuitionistic fuzzy number

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Abstract— This paper provides an algorithm for solving a multi-objective Intuitionistic fuzzy linear programming problem with data as triangular intuitionistic fuzzy numbers. Here, the multi-objective Linear programming problem is converted into a single objective linear programming and the problem is defuzzified using Triangular Intuitionistic fuzzy numbers. Then it is converted into equivalent crisp linear problems, and are solved using simplex method. A numerical example is provided to show the efficiency of the methodology.

Index Terms— Intuitionistic Fuzzy Sets,Triangular Intuitionistic fuzzy number,Membership and Nonmembership,Value,Ambiguity and Score value

1. Introduction

Linear Programming is one of the most important operations research techniques. It has been applied to solve many real world problems. An application of fuzzy optimization techniques to linear programming problems with multiple objectives has been presented by Zimmermann. It is extended to IFO by Attanassov.

The Intuitionistic fuzzy set (IFS) is an extension of fuzzy set (FS) where the degree of non-membership denoting the non-belongingness to a set is explicitly specified along with the degree of membership of belongingness to the set. Unlike the FS where the non-membership degree , in IFS , the membership and nonmebership degrees are more or less independent and related only by that the sum of two degrees must not exceed one.

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Recently, Li[1] has proposed a ratio ranking method for triangular intuitionistic fuzzy numbers. Then, it is applied to solve MADM problems. To this end, the value and ambiguity of TIFNs are used to obtain a new ranking approach. Using similar idea,Salahshour[6] proposed other new ranking approach for TIFNs based on the value and ambiguity. However, this approach is completely differ from Li's method. For ranking TIFNs, concert each TIFNs to the related TFNs based on its membership function and non-membership function. Then, for each obtained TFN we applied a new defuzzification to derive a real value related to the original TFN.

The organization of the paper is as follows.Some basic definitions and properties of Triangular Intuitionistic fuzzy numbers relevant to the present work and a new ranking function are given in section 2. In section 3, an algorithm is proposed to solve the multi-objective Intuitionistic linear programming problem. A numerical example is provided in section 4 and the paper is summarized in section 5.

2.Preliminaries

Definition:1

An **Intuitionistic fuzzy sets** (IFS) $\bar{a}$ assigns to each element x of the universe X a membership degree $\mu_{\bar{a}}(x)\epsilon[0,1]$ and a non-membership $\nu_{\bar{a}}(x)\epsilon[0,1]$ such that $\mu_{\bar{a}}(x)+\nu_{\bar{a}}(x)\leq 1$. An IFS $\bar{a}$ is mathematically, represented as $\{< x, \mu_{\bar{a}}(x), \nu_{\bar{a}}(x) >/x \in X\}$.

The value$\pi_{\bar{a}}(x) = (-\ \mu_{\bar{a}}(x)) - \nu_{\bar{a}}(x)$ is called the degree of **hesistancy** or the **intuitionistic index** of x to $\bar{a}$.

Definition:2

A TIFN $\bar{a} = \{\left(\underline{a}^{\mu}, a, \bar{a}^{\mu}, w_{\bar{a}}\right), \ (\underline{a}^{\nu}, a, \bar{a}^{\nu}, u_{\bar{a}})\}$is an IFS in ,whose **membership** and **non-membership** functions are respectively defined as,

$$\mu_a(x)= \begin{cases} (x - \underline{a}^{\mu})w_{\bar{a}}, & \underline{a}^{\mu} \leq x \leq a \\ w_{\bar{a}} & x = a \\ \frac{(\bar{a}^{\mu} - x)w_{\bar{a}}}{(\bar{a}^{\mu} - x)w_{\bar{a}}}, & a < x \leq \bar{a}^{\mu} \\ 0, & \text{otherwise} \end{cases}$$

$$\vartheta_{\bar{a}}(x)= \begin{cases} a - x + u_{\bar{a}}(x - \underline{a}^{\vartheta}), & \underline{a}^{\vartheta} \leq x \leq a \\ u_{\bar{a}}\ , & x = a \\ \frac{x - a + u_{\bar{a}}(\bar{a}^{\vartheta} - x)}{\bar{a}^{\vartheta} - a}, & a < x \leq \bar{a}^{\vartheta} \\ 0, & \text{otherwise} \end{cases}$$

The values $w_{\bar{a}}$ and $u_{\bar{a}}$ respectively represent the maximum degree of the membership and non-membership such that $0 \leq w_{\bar{a}} \leq 1$, $0 \leq u_{\bar{a}} \leq 1$ and $0 \leq w_{\bar{a}} + u_{\bar{a}} \leq 1$.

The same is depicted in Fig 1,

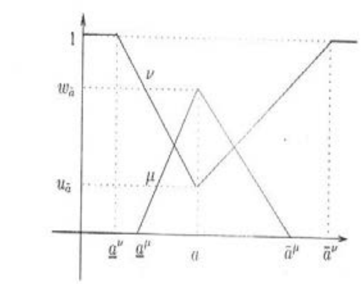

Figure 1: Triangular Intuitionistic Fuzzy Number (TIFN)

Definition: 3

Let $\bar{a} = \{\left(\underline{a}^{\mu}, a, \bar{a}^{\mu}, w_{\bar{a}}\right), \left(\underline{a}^{\vartheta}, a, \bar{a}^{\nu}, u_{\bar{a}}\right)\}$ be a TIFN. Then the **value** and **ambiguity** of $\bar{a}$ are given as follows.

(i) The **value of the membership function** of $\bar{a}$ is ,

$$V_{\mu}(\bar{a}) = \frac{\left(\underline{a}^{\mu} + 4a + \bar{a}^{\mu}\right)w_{\bar{a}}}{6}$$

While the **value of the non-membership function** is,

$$V_{\vartheta}(\bar{a}) = \frac{\left(\underline{a}^{\vartheta} + 4a + \bar{a}^{\vartheta}\right)(1 - u_{\bar{a}})}{6}$$

(ii) The **ambiguity of the membership function** of $\bar{a}$ is

$$A_{\mu}(\bar{a}) = \frac{(\bar{a}^{\mu} - \underline{a}^{\mu})w_{\bar{a}}}{3}$$

While the **ambiguity of the non-membership function** of $\bar{a}$ is

$$A_{\vartheta}(\bar{a}) = \frac{(\bar{a}^{\vartheta} - \underline{a}^{\vartheta})(1 - u_{\bar{a}})}{3}$$

3.The Proposed Algorithm

1. To find the solution of a multi-objective intuitionistic fuzzy linear programming problem, Define $\bar{a} = \{(\underline{a}^{\mu}, a, \bar{a}^{\mu}, w_{\bar{a}}), (\underline{a}^{\vartheta}, a, \bar{a}^{\nu}, u_{\bar{a}})\}$ and $\bar{b} = \{(\underline{b}^{\mu}, b, \bar{b}^{\mu}, w_{\bar{b}}), (\underline{b}^{\vartheta}, b, \bar{b}^{\nu}, u_{\bar{b}})\}$
2. Convert the multi-objective IFLP into a single objective IFLP using

$$\bar{a} + \bar{b} = \{(\underline{a}^{\mu} + \underline{b}^{\mu}, a + b, \bar{a}^{\mu} + \bar{b}^{\mu}; \min\{w_{\bar{a}}, w_{\bar{b}}\}),$$

$$(\underline{a}^{\vartheta} + \underline{b}^{\vartheta}, a + b, \bar{a}^{\vartheta} + \bar{b}^{\vartheta}; \min\{u_{\bar{a}}, u_{\bar{b}}\})\}$$

3. Defuzzify the IFLP into a crisp linear programming using

The **value of P($\tilde{a}$)** ,

$$P(\tilde{a}) = S_{\mu}(\tilde{a}) - S_{\nu}(\tilde{a})$$

Where, $S_{\mu}(\tilde{a}) = \frac{V_{\mu}(\tilde{a})}{1+A_{\mu}(\tilde{a})}$

$$S_{\nu}(\tilde{a}) = \frac{V_{\nu}(\tilde{a})}{1+A_{\nu}(\tilde{a})}$$

4. Formulate the linear programming problem.
5. Solve the LPP by any one of the used simplex procedure.

4. Numerical Example

$$\text{Max } \tilde{5}x_1 + \tilde{3}x_2$$

$$\text{Max } \widetilde{25}x_1 + \widetilde{48}x_2$$

Subject to

$$\tilde{4}x_1 + \tilde{3}x_2 \preccurlyeq \widetilde{12}$$

$$\tilde{1}x_1 + \tilde{3}x_2 \preccurlyeq \tilde{6}$$

$$x_1 + x_2 \geq 0$$

Where,

$C_1 = \tilde{5}$ = {(4,5,6;3/4)(4,5,6.1;1/4)}

$C_2 = \tilde{3}$ = {(2.5,3,3.2;1/2)(2,3,3.5;1/4)}

$P_1 = \widetilde{25}$ = {(19,25,33;0.9)(18,25,34;1)}

$P_2 = \widetilde{48}$ = {(44,48,54;0.9)(43,48,56;1)}

Subject to

$a_{11} = \tilde{4}$ = {(3,5,4.1;1)(3,4,5;0)}

$a_{12} = \tilde{3}$ = {(2.5.3.3.5;3/4)(2.4,3,3.6;1/5)}

$a_{21} = \tilde{1}$ = {(0,1,2;1)(0,1,2;0)}

$a_{22} = \tilde{3}$ = {(2.8,3,3.2;3/4)(2.5,3,3.2;1/6)}

$b_1 = \widetilde{12}$ = {(11,12,13;1)(11,12,14;0)}

$b_2 = \tilde{6}$ = {(5.5,6,7.5;3/4)(5,6,8.1;1/4)}

For, converting multi-objective into single objective,

$X_1=C_1+P_1$
={(4+19,5+25,6+33;min{3/4,0.9})(4+18,5+25,6.1+34;max{1/4,1})}

= {(23,30,39;3/4)(22,30,40.1;1)}

Similarly,

$X_2 = C_2 + P_2$ = {(46.5,51,57.2;0.5)(45,51,59.5;1)}

For,defuzzification,

X1 = {(23,30,39;3/4)(22,30,40.1;1)}

$V_{\mu}(\tilde{a})$ = 22.75 , $V_{\nu}(\tilde{a})$ = 0

$A_{\mu}(\tilde{a})$ = 4, $A_{\nu}(\tilde{a})$ = 0

$s_\mu(\tilde{a}) = 4.55,\ S_\nu(\tilde{a})$

Therefore, $P(X_1) = 4.55$

Similarly,

$P(X_2) = 9.2137$

$P(a_{11}) = 0.8775$

$P(a_{12}) = -0.018$

$P(a_{21}) = 0$

$P(a_{22}) = 0.005$

$P(b_1) = 3.14755$

$P(b_2) = 0.47073$

Therefore the crisp linear programming is

Max $4.55x_1 + 9.2137x_2$

Subject to $0.8775x_1 - 0.018x_2 \leq 3.14755$

$0x_1 + 0.005x_2 \leq 0.47033$

$x_1, x_2 \geq 0$

Using simplex method,the optimum feasible solution is obtained and is given by

$x_1 = 5.518$, $x_2 = 94.146$, Max Z = 892.53

Conclusion

This paper is useful to solve the multi-objective intuionistic linear programming Problem. It is also helps to defuzzification of TIFNs. An algorithm is proposed for solving mult-objective linear programming problem and a numerical example is given to show the efficiency of the methodology.

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